STAPLE

Physics 201 Name KEY

Exam 3 – Capacitance and Charges in Magnetic Fields

April 18, 2013

This is a closed book examination but during the exam you may refer to a 3"x5" note card with words of wisdom you have written on it. There is extra scratch paper available. Your explanation is worth ¾ of the points. Explain your answers!

A general reminder about problem solving:

- Show all your work.
- Really; Show All Work!
- Focus
    - Draw a picture of the problem
    - What is the question? What do you want to know?
    - List known and unknown quantities
    - List assumptions
- Physics
    - Determine approach – What physics principles will you use?
    - Pick a coordinate system
    - Simplify picture to a schematic (if needed)
- Plan
    - Divide problem into sub-problems
    - Modify schematic and coordinate system (if needed)
    - Write general equations
- Execute
    - Write equations with variables
    - Do you have sufficient equations to determine your unknowns?
    - Simplify and solve
- Evaluate
    - Check units
    - Why is answer reasonable?
    - Check limiting cases!

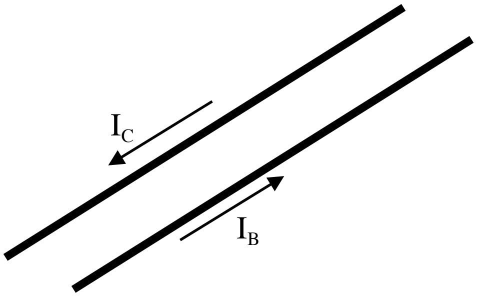
1. [4 PTS] You connect a resistor and capacitor in series with a battery. Initially the capacitor is uncharged. It takes 61 seconds for the voltage across the resistor to drop by one half the initial voltage. Hence the time constant is 88 seconds. You measured the resistance to be $R = 220\ \Omega$. What is the value of the capacitor?

    a) $C = 0.28\ F$
    [✓ b)] $C = 0.40\ F$
    c) $C = 0.68\ F$
    d) $C = 2.50\ F$
    e) $C = 3.61\ F$

Given $\tau = RC$ solve for the capacitance

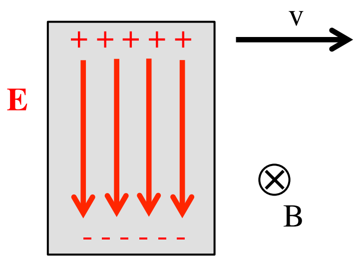
2. [4 PTS] Two very long wires, 10 cm apart, are hung parallel to each other. Current flows down each wire in opposite directions. Wire C has a current of 1.2 Amperes and wire B has a current of 0.6 Amperes.

    a) The two wires are attracted $\left|\vec{F_C}\right| = \frac{1}{4}\left|\vec{F_B}\right|$
    b) The two wires are attracted $\left|\vec{F_C}\right| = \frac{1}{2}\left|\vec{F_B}\right|$
    c) The two wires are attracted $\left|\vec{F_C}\right| = \left|\vec{F_B}\right|$
    [✓ d)] The two wires are repelled $\left|\vec{F_C}\right| = \left|\vec{F_B}\right|$
    e) The two wires are repelled $\left|\vec{F_C}\right| = 2\left|\vec{F_B}\right|$
    f) The two wires are repelled $\left|\vec{F_C}\right| = 4\left|\vec{F_B}\right|$



The force on wire C due to wire B will be equal in magnitude to the force on wire B due to wire C. Use Biot-Savart for a current carrying wire to find direction of magnetic field at wire B due to wire C. Find the force on wire B using $\vec{F} = Id\vec{l}\times\vec{B}$.

3. [4 PTS] Draw the resulting charge distribution for the metal rod traveling at a constant velocity (v) in a uniform magnetic field (B) shown below. Clearly indicated the resulting electric field if any.



The force on a charged particle moving in a magnetic field is $\vec{F} = q\vec{v}\times\vec{B}$ and $\vec{E}$ points in the direction from positive to negative charge

4. [4 PTS] An <u>electron</u> moving in the $+x$ direction enters a region of uniform magnetic field that is also oriented in the $+x$ direction. In which direction does the <u>electron</u> feel a force?

a) The $+y$ direction.
b) The $+z$ direction.
c) The $-y$ direction.
d) The $-z$ direction.
[✓ e)] There is no force.

The force on a charged particle moving in a magnetic field is $\vec{F} = q\vec{v}\times\vec{B}$. The cross product is zero for parallel vectors so the force is zero, $\hat{x}\times\hat{x} = 0$.

5. [4 PTS] A <u>neutron</u> moving in the $+x$ direction enters a region of uniform magnetic field that is oriented in the $+z$ direction. In which direction does the <u>neutron</u> feel a force?

a) The $+y$ direction.
b) The $+z$ direction.
c) The $-y$ direction.
d) The $-z$ direction.
[✓ e)] There is no force.

The force on a charged particle moving in a magnetic field is $\vec{F} = q\vec{v}\times\vec{B}$. The neutron is not charged so the force is zero. If this was a proton the force would be in the $\hat{x}\times\hat{z} = -\hat{y}$ direction.

6. [4 PTS] A resistor and capacitor are connected in series to a battery. You are told to double the time constant and are given an additional identical resistor and identical capacitor. Indicate which of the following actions will double the time constant.

a) Add a resistor in parallel to the circuit resistor.
[✓ b)] Add a resistor in series to the circuit resistor.
[✓ c)] Add a capacitor in parallel to the circuit capacitor
d) Add a capacitor in series to the circuit capacitor.
e) Add both a capacitor and resistor in parallel to their circuit counterparts.
f) Add both a capacitor and resistor in series to their circuit counterparts.

Need to double the resistance or capacitance. Resistors in series add. Capacitors in parallel add.

7. [4 PTS] A wire with current running through it is placed between the poles of a strong magnet. The wire is part of a simple battery and resistor circuit but is free to move. The magnetic field is in the +*y* direction and the wire is parallel to the *x*-axis. If the wire is deflected in the +*z* direction what is the direction of the current?
   a) There is no current but there is a net negative charge on the wire.
   b) There is no current but there is a net positive charge on the wire.
   c) The current is traveling in the –*x* direction.
   [✓ d)] The current is traveling in the +*x* direction.

The force on a current carrying wire is $\vec{F} = Id\vec{l}\times\vec{B}$. So current has to be in +*x* direction since $\hat{z} = \hat{x}\times\hat{y}$

8. [4 PTS] A charged object, $q = -10\mu C$, is moving through a region of space with a constant magnetic field, $\vec{B} =< 0,0,1.1 > T$. When the object is at $\vec{x} =< 0,0,2 > m$, it has a velocity $\vec{v} =< 3\times10^5, 4\times10^5, 0 > m/s$. Assume the object is nonrelativistic, $\gamma = 1$. What is the magnitude of the force on the object?
   a) $|\vec{F}| = 0\ N$
   b) $|\vec{F}| = 2.2\ N$
   [✓ c)] $|\vec{F}| = 5.5\ N$
   d) $|\vec{F}| = 11\ N$
   e) $|\vec{F}| = 13.2\ N$

The force on a charged particle moving in a magnetic field is $\vec{F} = q\vec{v}\times\vec{B}$. The velocity plane and magnetic field are at right angles so the magnitude of the force is $|\vec{F}| = |q||\vec{v}||\vec{B}| = (1\times10^{-5}C)\left(5\times10^5 \frac{m}{s}\right)(1.1\ T)$

9. [4 PTS] You connect three identical capacitors ($C = 3\ mF$) in series. What is the value of the effective (or total circuit) capacitance?
   a) $C = 9\ mF$
   b) $C = 6\ mF$
   [✓ c)] $C = 1\ mF$
   d) $C = \frac{1}{3}mF$
   e) $C = \frac{1}{9}mF$

Capacitors in series add $\frac{1}{C_{tot}} = \sum_i \frac{1}{C_i}$. So solve for the capacitance

10. [4 PTS] A region of space containing a net positive charge is enclosed by a virtual sphere to determine the electric field flux. If the radius of the enclosing virtual sphere is doubled (increasing the volume by 8 times and the area by 4 times), what is change in the flux?
   a) The flux increases by 4 times.
   b) The flux increases by 2 times.
   [✓ c)] The flux does not change.
   d) The flux decreases by 2 times.
   e) The flux decreases by 4 times.

$\Phi = \oint \vec{E}\cdot\hat{n}\,dA$ so increasing the radius by 2x increases the area by 4, but also decreases the electric field by 4 (since $\vec{E} = \frac{kq}{r^2}\hat{r}$) so the net result is no change in flux.

The next two problems can be done using problem solving sheets or on additional paper. Follow the problem solving guidelines.

11. [12 PTS] You connect a resistor and capacitor in series with a battery and a switch. The switch is initially open and the capacitor is discharged. At time t = 0 seconds you close the switch.
    a) Determine an equation for the power used by the capacitor as a function of time.
    b) Determine an equation for the power used by the resistor as a function of time.
    c) Compare these values to the power delivered by the battery as a function of time.
    d) [BONUS 4 PTS]: Integrate the power used by the capacitor to derive the energy stored in a capacitor.

Use $Q = C\Delta V$ and $\Delta V = IR$. The voltage across the resistor decreases exponentially, $V_R = V_B e^{-t/\tau}$, while the voltage across the capacitor increases exponentially, $V_C = V_B\left(1 - e^{-t/\tau}\right)$, where $\tau = RC$. The current in the series circuit is $I = \frac{V_B}{R} e^{-t/\tau}$. Since $P = IV$, then you can write $P_C = \frac{(V_B)^2}{R} e^{-t/\tau}\left(1 - e^{-t/\tau}\right)$ and $P_R = \frac{(V_B)^2}{R} e^{-2t/\tau}$ both of which approach zero as time increases. The voltage across the battery is constant but the current draw decreases so $P_B = \frac{(V_B)^2}{R} e^{-t/\tau}$which is equal to the sum $P_B = P_R + P_C$. To find the energy in the capacitor integrate over all time,

$$E_C = \int_{t=0}^{t=\infty} P_C\, dt = \int_0^\infty \frac{(V_B)^2}{R} e^{-t/\tau}\left(1 - e^{-t/\tau}\right) dt = \frac{(V_B)^2}{R}\left(\int_0^\infty e^{-t/\tau} dt - \int_0^\infty e^{-2t/\tau} dt\right) = \frac{(V_B)^2}{R}\left(-\tau e^{-t/\tau}\Big|_0^\infty + \frac{\tau}{2} e^{-2t/\tau}\Big|_0^\infty\right) = \frac{(V_B)^2}{R}\left(\tau - \frac{\tau}{2}\right) = \frac{(V_B)^2\tau}{2R} = \frac{(V_B)^2(RC)}{2R} = \frac{1}{2}(V_B)^2 C = \frac{1}{2} Q V_B$$

12. [12 PT] You are designing a cyclotron to accelerate alpha particles (doubly ionized Helium atoms). Assume neutrons have approximately the same mass as protons.
    a) Derive the equation for the oscillation frequency (angular speed) for this particle at low speeds.
    b) Determine the actual angular speed for a 3.0 Tesla uniform magnetic field.
    c) Compare the oscillation frequency of alpha particles to protons.

The oscillation frequency is derived by setting the time rate of change of the momentum equal to the mass times the centripetal acceleration, $a_c = \frac{v^2}{r}$ (since the object is in circular motion). The time rate of change of the momentum is equal to the net force which is $\vec{F} = q\vec{v}\times\vec{B}$. Solve for the ratio of velocity divided by radius to determine the oscillation frequency, $\omega = \frac{qB}{m}$. The oscillation frequency is ½ that for a proton.

$\frac{\mu_0}{4\pi} = 10^{-7}\frac{N}{A^2}$

$\frac{1}{4\pi\epsilon_0} = k = 9\times10^{9}\frac{Nm^2}{C^2}$

mass of electron $m_e = 9.109\times10^{-31}kg$

charge of electron $q_e = 1.602\times10^{-19}C$

charge of neutron $q_n = 0\ C$

$V_{sphere} = \frac{4\pi r^3}{3}$ and $A_{sphere} = 4\pi r^2$

mass of proton $m_p = 1.673\times10^{-27}kg$

charge of proton $q_p = 1.602\times10^{-19}C$

mass of neutron $m_n = 1.675\times10^{-27}kg$